ASL Exercise 6

2K Factorial Designs

How do you determine the effect of k factors on a system?

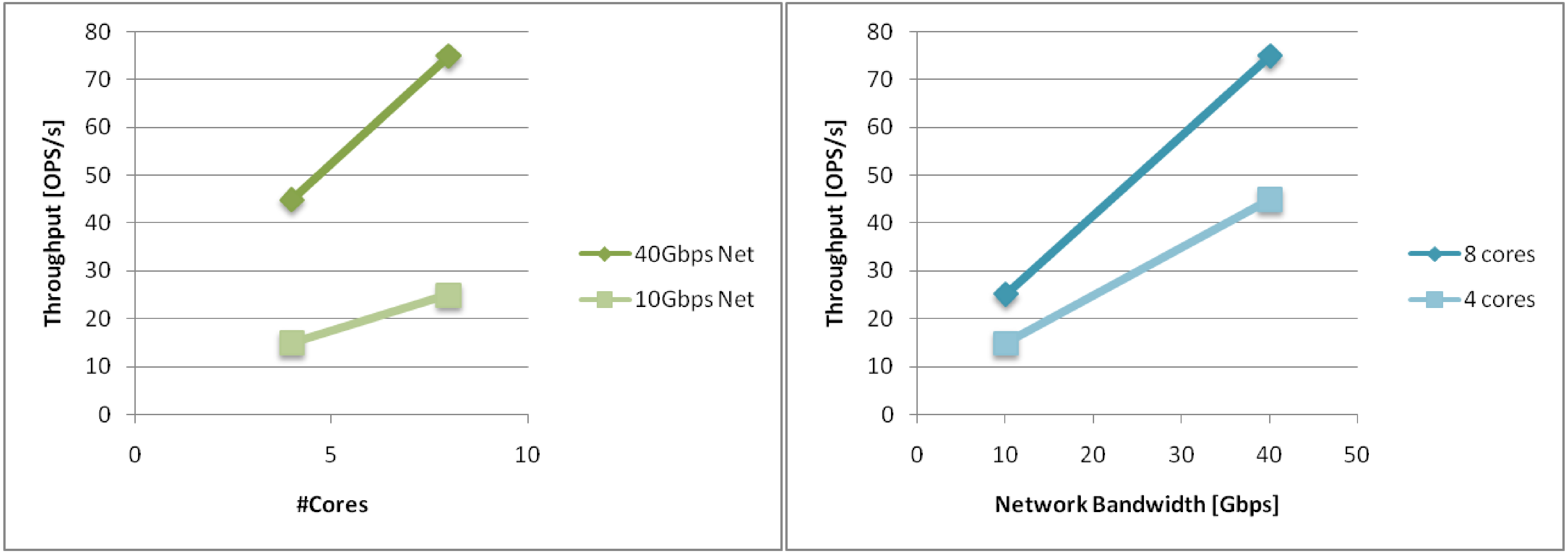
- Design experiments, where each factor can have 2 values.
- Perform 2^k experiments.
- Analyze the results, giving insights on:
   1. The effect of factors alone on the result.
   2. The contributed effect of factor together on the result.

Example:

Factor A:	Network Bandwidth	Values: 10 Gbit/s, 40 Gbit/s
Factor B:	# CPU Cores	Values: 4, 8

Measure throughput (requests/second)

What if we plot it?



- Difficult to look at two graphs at the same time
- Hard to reach definitive conclusion

Use a table-based representation instead:

Factor A: Network Bandwidth Values: 10 Gbit/s, 40 Gbit/s

Factor B: # CPU Cores Values: 4, 8

Measure throughput (requests/second)

Define 2 variables: xA, xB

$$xA = \begin{cases} -1 & \text{if 10 Gbit/s} \\ 1 & \text{if 40 Gbit/s} \end{cases} \qquad xB = \begin{cases} -1 & \text{if 4 Cores} \\ 1 & \text{if 8 Cores} \end{cases}$$

Example:

Experiment	xA (bandwidth)	xB (# cores)	Y (TPUT)
1	-1 (10 Gbit/s)	-1 (4 cores)	15
2	1 (40 Gbit/s)	-1 (4 cores)	45
3	-1 (10 Gbit/s)	1 (8 cores)	25
4	1 (40 Gbit/s)	1 (8 cores)	75

Example:

Define 2 variables: xA, xB

$$xA = \begin{cases} -1 & \text{if 10 Gbit/s} \\ 1 & \text{if 40 Gbit/s} \end{cases} \qquad xB = \begin{cases} -1 & \text{if 4 Cores} \\ 1 & \text{if 8 Cores} \end{cases}$$

Solve the equation:

$$y = q0 + qA*xA + qB*xB + qAB*xA*xB$$

y: throughput

qA: Effect of bandwidth

qB: Effect of # Cores

qAB: Combined Effect

Example:

Experiment	xA (bandwidth)	xB(# cores)	y
1	-1 (10 Gbit/s)	-1 (4 cores)	y1
2	1 (40 Gbit/s)	-1 (4 cores)	y2
3	-1 (10 Gbit/s)	1 (8 cores)	y3
4	1 (40 Gbit/s)	1 (8 cores)	y4

Measured Values

$$y = q0 + qA*xA + qB*xB + qAB*xA*xB$$

y: throughput

qA: Effect of bandwidth

qB: Effect of # Cores

qAB: Combined Effect

Example:

I	xA	xB	xA*xB	y
1	-1	-1	1	15
1	1	-1	-1	45
1	-1	1	-1	25
1	1	1	1	75
40	20	10	5	Total/4

So-called Effects!

y = q0 + qA*xA + qB*xB + qAB*xA*xB

Allocation of Variation

- Importance of a factor: Proportion of the total variation in the result that is explained by the factor.
- Total variation of y = SST= $\sum_{i=1}^{2^2}(y_i - \bar{y})^2$
- For a 2^2 design, SST= 2^2(qA^2 + qB^2 + qAB^2)

Variation explained by A

Variation explained by B

Variation explained by
The interaction of A and B

Example showing the allocation of variation

Experiment	xA (bandwidth)	xB(# cores)	Y (Throughput)
1	-1 (10 Gbit/s)	-1 (4 cores)	15
2	1 (40 Gbit/s)	-1 (4 cores)	45
3	-1 (10 Gbit/s)	1 (8 cores)	25
4	1 (40 Gbit/s)	1 (8 cores)	75

SST = 2^2(20^2 + 10^2 + 5^2) = 2100

SSA = 2^2(20^2) = 1600

SSB = 2^2(10^2) = 400

SSAB = 2^2(5^2) = 100

SSA/SST

SSB/SST

SSAB/SST

Parameter	Mean Estimate	Variation Explained (%)
q0	40	
qA	20	76 %
qB	10	19 %
qAB	5	5 %

Interpretation

Parameter	Mean Estimate	Variation Explained (%)
q0	40	
qA	20	76 %
qB	10	19 %
qAB	5	5 %

- Average throughput is 40 reqs/second
- The throughput is mostly affected by the network bandwidth: 76 % of the variation
- The number of cores contributes 19 % to the variation
- The interactive effect of network bandwidth and number of cores is relatively less: only 5 %

How about replication: $2^k r$ Factorial Designs

Isolate experimental errors!

Solve the equation:

y = q0 + qA*xA + qB*xB + qAB*xA*xB + e

Experimental error!

Example:

y – y_mean

I	xA	xB	xA*xB	y	y_mean
1	-1	-1	1	(15,18,12)	15
1	1	-1	-1	(45,48,51)	48
1	-1	1	-1	(25,28,19)	24
1	1	1	1	(75,75,81)	77
41	21.5	9.5	5		Total/4

i	e1	e2	e3
1	0	3	-3
2	-3	0	3
3	1	4	-5
4	-2	-2	4

Sum of squared errors: $SSE = \sum_{i=1}^{2^k} \sum_{j=1}^{r} e_{ij}^2$

SSE = 102

Allocation of variation:

SST = SSA + SSB + SSAB + SSE

$$SST = 2^2 r \cdot qA^2 + 2^2 r \cdot qB^2 + 2^2 r \cdot qAB^2 + \sum_{i}^{2^2} \sum_{j}^{r} e_{ij}^2$$

What if the effects are multiplicative?

- Until now we have assumed the model:

$$y = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B + e$$

All effects are added!

- If the effects multiply (for example, A: size of a workload and B: clock frequency)
- Compute in log domain!

$$y = A * B$$

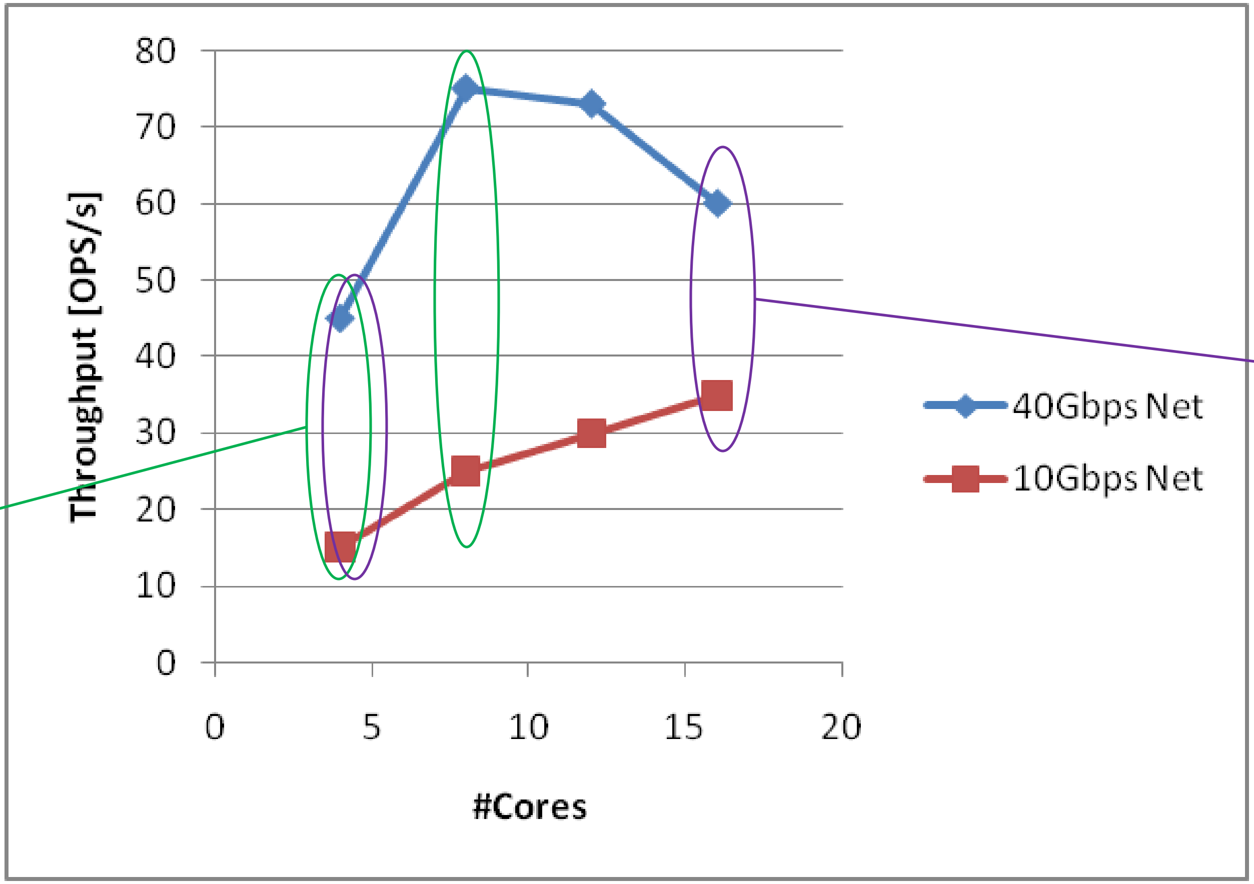
$$\downarrow$$

$$\log(y) = \log(A) + \log(B)$$

- After taking the log of the measurements, you can use the model exactly as before.

Discussion

- What happens if we do a 2k on the following graph? Which points to choose?



Conclusion could be that choice of network has much higher effect

Conclusion could be that both have equal effect